Producing massive sterile neutrinos as warm Dark Matter

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Abstract

Cosmological production of sterile neutrinos mixed with active ones is recalculated with the exact form of the coherence breaking terms in the density matrix. The results differ by approximately a factor 4 with respect to earlier simplified calculations. Sterile neutrinos remain viable candidates for warm dark matter particles.

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1 Introduction

There is increasing evidence that the dark matter particle has a mass about 1 keV. First of all N-body simulations of large scale structure seem to produce too many satellite galaxies in comparison with observations, a problem which is solved by reducing power on small scales [1]. This is most easily achieved through free streaming with a particle with a mass about keV. Furthermore, disk galaxies are formed without the need for stellar feedback if the dark matter has mass about keV [2]. On the other hand, to reproduce the observed properties of Lyman-α clouds one gets a lower bound on the mass of the dark matter particle [3], and the existence of a massive black hole at large redshift also gives a lower bound on the dark matter free streaming mass [4]. All these studies point towards the mass of the dark matter particle near 1 keV.

There are several interesting candidates for being the dark matter (for recent overviews see refs. [2, 5]), and the candidate preferred by minimalists is naturally a neutrino, being the only non-baryonic DM candidate known to exist. The simplest case is, if the DM is an active neutrino (ν_e, ν_μ or ν_τ). BBN can only exclude active neutrino masses bigger

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than about 0.3 MeV [6], however, if the neutrino was in equilibrium in the early universe, then it must decay in order not to overclose the universe [7]. Another possibility is to avoid thermalizing the massive neutrino, which e.g. can be achieved by a low reheat temperature [8].

To give the neutrinos a mass one must extend the standard model slightly, and a suggestive possibility then appears, namely to introduce right handed or sterile neutrinos. Such sterile neutrinos could naturally mix with the active neutrinos in the same way as the active neutrinos probably mix among each other, and if the mixing angle is small enough then it would be difficult to discover the sterile neutrinos in terrestrial experiments. We will in this paper consider such sterile neutrinos, and we take both the mass and the (small) mixing angle as free parameters.

2 The physics behind the production

The production rate of active neutrinos is $\Gamma/H = (T/T_W)^3$ where H is the Hubble expansion parameter, T is the temperature of the plasma, and $T_W \approx 3$MeV is approximately the freeze out temperature of the active neutrinos. For sterile neutrinos one often approximates the production rate by [9, 10, 11]

$$\frac{\Gamma}{H} = \frac{\sin^2 2\theta_M}{2} \left(\frac{T}{T_W}\right)^3 , \tag{1}$$

where θ_M is the mixing angle in matter. If the mixing angle in matter was the same as in vacuum, then there would always have been a (high) temperature, where $\Gamma/H \sim 1$, and hence the sterile neutrino would have been in equilibrium. This is not the case, however, because the mixing angle in matter differs from the vacuum one at high temperatures as [12]

$$\sin 2\theta_M = \frac{\sin 2\theta}{1 + 3.73 \cdot 10^{-20}\, C_l\, m(\mathrm{MeV})^{-2}\, (y^2/x^6)} , \tag{2}$$

where the ν_s mass, m, is measured in MeV and we used the expansion parameter of the universe, a, to introduce the variables

$$x = 1\,\mathrm{MeV}\, a, \quad y = Ea\, , \tag{3}$$

and neglected a possible entropy release so that the temperature drops according to $T = 1/a$. The numerical coefficient C_l depends upon the neutrino flavour: $C_l = 0.61$ for ν_e and $C_l = 0.17$ for ν_τ and ν_μ. However, for the temperatures close to or above the muon mass C_l becomes the same for ν_e and ν_μ. It is therefore clear, that the production rate at very high temperature is strongly suppressed, $\Gamma/H \sim T^{-12}$. Since one doesn't expect any sterile neutrinos to be produced at the end of inflation, $f_{in} = 0$, one can integrate the Boltzmann equation in time to find the abundance of sterile neutrinos [13]

$$Hx\,\partial_x f_s = \frac{1}{2}\sin^2 2\theta_M\,\Gamma_W\,f_a\,. \tag{4}$$

One of the difficult questions in this approach is what to use for Γ_W? Should one include only annihilation processes, both annihilation and scattering or some combination? This question was recently addressed in [14] where bounds on light ν_s were found from BBN, and we will here apply those results to sterile neutrino production with the focus on keV masses. Let us already now mention, that the effect of treating this problem correctly differs significantly (a factor of about 4) from the simple approach of assuming $\Gamma_W = \Gamma_{el} + \Gamma_{ann}$.

Before starting let us clarify that we consider the non-resonant case with $m_{\nu_s} > m_{\nu_\alpha}$, and since observations indicate that the active neutrino masses are small (for a recent overview see e.g. [15]) we often simply write $\sqrt{\delta m^2} = m$.

3 Kinetic equations

The kinetic equations describing oscillating neutrinos in the early universe with the proper account of coherence breaking terms were derived in [16, 17]. The equations are nonlinear and contain multidimensional integrals over neutrino phase space. Nevertheless, assuming kinetic, though not chemical equilibrium, for the active-active component, ρ_{aa}, of the neutrino density matrix (see eq. (22) below) and taking the simplified approach of Boltzmann statistics will allow us to solve the equations analytically. To set notation let us follow and expand the analysis of ref. [14].

The kinetic equations describing the evolution of the density matrix, ρ, are

$$\begin{aligned}
\dot\rho_{aa}(p_1) &= -FI - \int d\Pi A_{el}^2 \left[\rho_{aa}(p_1) f_l(p_2) - \rho_{aa}(p_3) f_l(p_4)\right] \\
&\quad - \int d\Pi A_{ann}^2 \left[\rho_{aa}(p_1)\bar\rho_{aa}(p_2) - f_l(p_3) f_{\bar l}(p_4)\right], \qquad (5)\\
\dot\rho_{ss}(p_1) &= FI, \qquad (6)\\
\dot R(p_1) &= WI - \int d\Pi (A_{el}^2/2) \left[R(p_1) f_l(p_2) - R(p_3) f_l(p_4)\right] \\
&\quad - \int d\Pi (A_{ann}^2/4) \left[\rho_{aa}(p_1)\bar R(p_2) + \bar\rho_{aa}(p_2) R(p_1)\right], \qquad (7)\\
\dot I(p_1) &= -WR - (F/2)\left(\rho_{ss} - \rho_{aa}\right) - \int d\Pi (A_{el}^2/2) \left[I(p_1) f_l(p_2)\right. \\
&\quad \left. - I(p_3) f_l(p_4)\right] - \int d\Pi (A_{ann}^2/4) \left[\rho_{aa}(p_1)\bar I(p_2) + \bar\rho_{aa}(p_2) I(p_1)\right]. \qquad (8)
\end{aligned}$$

where $\dot\rho = (\partial_t - Hp\partial_p)\rho$, and $\int d\Pi$ is a short notation for the integration over all momenta except the first one

$$\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4\left(p_1+p_2-p_3-p_4\right), \qquad (9)$$

R and I are the real and imaginary parts of the off diagonal matrix elements

$$\rho_{as} = \rho_{sa}^* = R + iI, \qquad (10)$$

and we have defined

$$\begin{aligned}
F &= \delta m^2 \sin 2\theta / 2E, \qquad (11)\\
W &= \delta m^2 \cos 2\theta / 2E + C_l (G_F^2 T^4 E/\alpha). \qquad (12)
\end{aligned}$$

An important quantity is $F/W = \tan 2\theta/(1+\epsilon(y,\delta m^2)T^6)$ with the rescaled momentum $y = p/T$, which plays the role of the matter mixing angle (compare to eq. (2)).

Since the elastic scattering terms disappear after integrating over p_1, it is clear from eqs. (5, 6) that the lepton number, $n_s + n_a \sim \int d^3p_1\,(\rho_{ss}+\rho_{aa})$ is constant if $A_{ann} = 0$

$$\dot n_s + \dot n_a \sim \int d\Pi \int d^3p_1\, A_{ann}^2, \qquad (13)$$

which indicates that the effect of oscillations in the total number density vanishes when $A_{ann} = 0$. It means in particular that a creation of ν_s species does not have any effect on BBN because it is accompanied by an equal reduction of the number density of active neutrinos [14]. This is of course only true if $m_{\nu_s} \ll 1$ MeV. We are, however, interested in the individual value of the sterile neutrino number density, n_s, because its magnitude determines the energy density of WDM today, if ν_s indeed constitute the dark matter. To calculate this quantity both annihilation and elastic scattering are essential.

First, we will formally solve eqs. (7) and (8) to express the non-diagonal components R and I through the diagonal ones. To this end we approximate these equations as

$$\begin{aligned} \dot{R} &= WI - \gamma R\,, & (14)\\ \dot{I} &= -WR + \frac{F}{2}\left(\rho_{aa} - \rho_{ss}\right) - \gamma I\,. & (15) \end{aligned}$$

In fact the aim of this paper is to proceed beyond this approximation and to describe the coherence breaking by the exact expressions given by eqs. (5-8) instead of the generally used simplified anticommutator

$$\dot{\rho} = ... - \{\Gamma, \rho - \rho_{eq}\}\,, \tag{16}$$

where Γ is the 2×2-matrix with the only non-zero entry $\Gamma_{aa} = \gamma$. An essential point is that the expression for R does not depend on γ (see eq. (21) below) in the limit that the oscillation frequency W is much larger than γ. In this limit one finds the same expression for R with any form of the coherence breaking terms.

The formal solution of eqs. (14, 15) is

$$\begin{aligned} I &= \int_0^q dq_1 e^{-\Delta\Gamma} \cos\Delta\Phi\, \frac{F}{2}\left(\rho_{aa} - \rho_{ss}\right)\,, & (17)\\ R &= \int_0^q dq_1 e^{-\Delta\Gamma} \sin\Delta\Phi\, \frac{F}{2}\left(\rho_{aa} - \rho_{ss}\right)\,, & (18) \end{aligned}$$

where we have followed [18] and introduced $q = \xi_a x^3$ (one has $\xi_e \approx 6.6 \cdot 10^6\,(m/\mathrm{keV})$ and $\xi_\mu \approx 1.3 \cdot 10^7\,(m/\mathrm{keV})$) with $x = 1\mathrm{MeV}\,a(t)$, where $a(t)$ is the cosmological scale factor. We have defined $\Delta\Gamma = \Gamma(q, y) - \Gamma(q_1, y)$, $\Delta\Phi = \Phi(q, y) - \Phi(q_1, y)$, and Φ and Γ obey [19]

$$\partial_q \Phi = \frac{K}{y} W \quad \text{and} \quad \partial_q \Gamma = \frac{K}{y}\gamma\,, \tag{19}$$

with

$$K_e = 5.63 \cdot 10^7\, m\,(\cos 2\theta)^{1/2} \quad \text{and} \quad K_{\mu,\tau} = 2.97 \cdot 10^7\, m\,(\cos 2\theta)^{1/2}\,, \tag{20}$$

where m is in keV. The form of γ has been debated in the literature, where [9] argued that $\gamma = \gamma_{ann}$, hence including only annihilation, whereas [20] argued that $\gamma = \gamma_{ann} + \gamma_{el}$, hence including both annihilation and elastic scattering. The difference between these two approaches is about 2 orders of magnitude in the exclusion plots for BBN, and the exact treatment of the collision terms [14] found the result to be somewhere in between. We will here follow the approach of ref. [14] where no ambiguity arises.

The bulk of sterile neutrinos is produced at rather high temperatures, $T_s \approx 10^2$ MeV $(m/\text{keV})^{1/3}$ [9, 10, 13, 21], as can be estimated from the approximate expressions (1, 2). In this temperature range the coherence breaking is very fast and the integral over q_1 in eqs. (17, 18) sits near the upper limit $q_1 = q$. Thus one finds $R \approx F/2\,(\rho_{aa} - \rho_{ss})\,W/(W^2 + \gamma^2)$, and since we are considering the non-resonant case we have $W >> \gamma$, and for keV particles we have $\rho_{aa} >> \rho_{ss}$, and we finally get

$$R = \frac{F}{2W}\rho_{aa}\,. \tag{21}$$

As we have already mentioned the result is independent of γ.

We will assume in what follows that the active neutrinos are kept in kinetic equilibrium with a non-zero effective chemical potential, that have equal values for ν_a and $\bar{\nu}_a$

$$\rho_{aa} = f_{eq}(y, \xi) = \left[\exp(y - \xi)\right]^{-1} \approx \exp\left[-y + \xi\right]\,. \tag{22}$$

This is a very good approximation at the temperatures where sterile neutrinos were produced, $T \sim 10^2$ MeV. Kinetic equilibrium is better maintained than the chemical one because the elastic scattering is approximately an order of magnitude stronger than the annihilation.

What will happen below is the following. When inserting the solution (21) in equation (7) for $\dot{R}$, we can isolate I. One can then insert that expression for I in the equations (5, 6) for ρ_{aa} and ρ_{ss}, which then can be integrated over p_1. By the integration over p_1 the normal elastic scattering terms on the rhs of eq. (5) naturally disappear, but the scattering terms in the expression for I have momentum dependence which do not cancel.

Let us now treat equation (6) for ρ_{ss} as just described, that is, inserting the formal solution for R, eq. (21) into eq. (7) gives

$$I = \frac{1}{W}\left[\dot{R} + \text{collision integral}\right] . \tag{23}$$

This expression for I can be inserted in $\dot{\rho}_{ss} = FI$. Since we changed from time to x, the lhs becomes $d/dt \to xH\, d/dx$. When we integrate over momentum p_1 we find

$$\pi^2 xH \frac{dn_s}{dx} = \int d^3p_1 \, \frac{F}{W}\left(\dot{R} + \text{collision integral}\right) . \tag{24}$$

The rhs is then found to be

$$\frac{K_l}{288x^4}\left[\left(6I_2 + I_1^2\right) + \kappa\left(6I_2 - I_1^2\right)\right] + \partial_x\left(\int dy\, \frac{y^2 e^{-y}}{8}\left(\frac{F}{W}\right)^2\right) , \tag{25}$$

with $K = 8G_L^2(1+2(g_L^2+g_R^2))/(\pi^3 H x^2)$, $\kappa = (18+16(g_L^2+g_R^2))/(1+2(g_L^2+g_R^2))$ and $I_n = \int_0^\infty dy\, y^3 e^{-y}(F/W)^n$. Eq. (24) can now be integrated over x and one finds

$$n_s^e = \frac{\tan^2 2\theta}{\pi^2}\left(\frac{1}{4} + 4.4\cdot 10^4\left(\frac{m}{\text{keV}}\right)\right) , \tag{26}$$

$$n_s^{\mu,\tau} = \frac{\tan^2 2\theta}{\pi^2}\left(\frac{1}{4} + 5.5\cdot 10^4\left(\frac{m}{\text{keV}}\right)\right) , \tag{27}$$

which is the main result of this paper. This can readily be compared with eq. (9) of ref. [21], where the simplifying assumption $\gamma = \gamma_{el} + \gamma_{ann}$ lead to

$$n_s^e = \frac{\sin^2 2\theta}{\pi^2}\left(1.8\cdot 10^5\left(\frac{m}{\text{keV}}\right)\right) , \tag{28}$$

$$n_s^{\mu,\tau} = \frac{\sin^2 2\theta}{\pi^2}\left(2.5\cdot 10^5\left(\frac{m}{\text{keV}}\right)\right) , \tag{29}$$

and we therefore see, that the importance of considering the correct Boltzmann equations is more than a factor 4 in the relation between the mixing angle and the mass.

If the sterile neutrinos indeed constitute the dark matter, then their number density is easily found by using $\rho_s = \Omega_{DM} 10\, h^2\, \text{keV/cm}^3$, which gives us

$$\frac{n_s}{n_e} = 1.27\cdot 10^{-2}\left(\frac{\text{keV}}{m}\right)\left(\frac{\Omega_{DM}}{0.3}\right)\left(\frac{h}{0.65}\right)^2 . \tag{30}$$

Thus we find by comparing eqs. (26, 27) with eq. (30) that the necessary values of mass/mixing are

$$\sin^2 2\theta_{se} \approx 2.9\cdot 10^{-7} m(\text{keV})^{-2} \quad \text{and} \quad \sin^2 2\theta_{s\mu} \approx 2.3\cdot 10^{-7} m(\text{keV})^{-2} , \tag{31}$$

which upon comparison with the diffuse photon background (see details and references in ref. [21]) gives approximately a factor 2 stronger upper bounds on the mass, about 19 keV for mixing with ν_e, and 21 keV for mixing with $\nu_{\mu,\tau}$. Naturally lower bounds on the mass exist, e.g. if BBN allows 0.3 extra neutrinos around $T \sim$ MeV, then from eq. (30) one gets $m > 42$ eV with $\Omega_{DM} = 0.3$ and $h = 0.65$.

Recently there appeared the paper [22] where similar problems were addressed both for resonance and non-resonance cases. The number density of sterile neutrinos obtained in that paper for non-resonance case is $\Omega_\nu h^2 = 0.3\,(\sin 2\theta/10^{-5})^2\,(m/100\text{keV})^2$. It is about 2 times larger than our result. The difference might be attributed to a different treatment of the coherence breaking terms in ref. [22] and in the present work. However, the results of ref. [22] also differs from our earlier paper [21], where the same approximation, eq. (16), for coherence breaking was taken. In view of the potential importance of sterile neutrinos as warm dark matter particles, it is desirable to resolve this discrepancy.

4 Conclusion

We have reconsidered our previous calculations [21] (as well as similar calculations of other papers) of the cosmological production of sterile neutrinos, ν_s, mixed with active ones (ν_e, ν_μ, or ν_τ). Instead of an approximate treatment of the coherence breaking terms as given by eq. (16), used in previous calculations, we use the approach of ref. [14], where the coherence breaking was treated exactly.

The correct treatment of the collision terms results in approximately 4 times smaller cosmological number density of sterile neutrinos with keV mass. Hence twice larger mixing angle is necessary to produce the same mass density of warm dark matter.

We thus see, that a sterile neutrino with mass $\lesssim 20$ keV, and mixing angle given by eq. (31), remains a simple and promising warm dark matter candidate.

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References

[1] P. Colin, V. Avila-Reese and O. Valenzuela, Astrophys. J. **542** (2000) 622.

[2] J. Sommer-Larsen and A.D. Dolgov, [astro-ph/9912166].

[3] V. Narayanan, D. N. Spergel, C.-P. Ma, R. Davé, [astro-ph/0005095].

[4] R. Barkana, Z. Haiman, J. P. Ostriker, [astro-ph/0102304].

[5] P. Bode, J. P. Ostriker, N. Turok, [astro-ph/0010389].

[6] A. D. Dolgov, S. H. Hansen and D. V. Semikoz, Nucl. Phys. B **524** (1998) 621 [hep-ph/9712284].

[7] S.S. Gerstein, Ya.B. Zeldovich, ZhETF Pis'ma Red. **4** (1966) 174;
R. Cowsik and J. McClelland, Phys. Rev. Lett. **29** (1972) 669.

[8] G. F. Giudice, E. W. Kolb, A. Riotto, D. V. Semikoz, I. I. Tkachev, [hep-ph/0012317].

[9] R. Barbieri and A. Dolgov, Phys. Lett. **B237** (1990) 440;
R. Barbieri and A. Dolgov, Nucl. Phys. **B237** (1991) 742.

[10] K. Kainulainen, Phys. Lett. **B244** (1990) 191.

[11] K. Enqvist, K. Kainulainen, and M.Thomson, Nucl. Phys. **B373** (1992) 498.

[12] D. Nötzold and G. Raffelt, Nucl. Phys. **B307** (1988) 924.

[13] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. **72** (1994) 17.

[14] A. D. Dolgov, Phys. Lett. B (to be published), [hep-ph/0006103].

[15] J. Ellis, Nucl. Phys. Proc. Suppl. **91** (2000) 503 [hep-ph/0008334].

[16] A.D.Dolgov, Yad. Fiz. **33** (1981) 1309; English translation: Sov. J. Nucl. Phys. **33** (1981) 700.

[17] G. Sigl and G. Raffelt, Nucl.Phys. **B406** (1993) 423.

[18] A. D. Dolgov, S. H. Hansen, S. Pastor and D. V. Semikoz, Astropart. Phys. **14** (2000) 79, [hep-ph/9910444].

[19] A. D. Dolgov, [hep-ph/0102125].

[20] K. Kainulainen, Phys. Lett **B244** (1990) 191;
K. Enqvist, K. Kainulainen and M. Thomson, Nucl. Phys. **B373** (1992) 498;
X. Shi, D. Schramm and B. Fields, Phys. Rev. **D48** (1993) 2563.

[21] A. D. Dolgov and S. H. Hansen, Astropart. Phys. (to be published), [hep-ph/0009083].

[22] K. Abazajian, G.M. Fuller, M. Patel, [astro-ph/0101524].